1. **K-theory**

1.1. **∞-categories.**
   (1) Reminder on simplicial stuff
   (2) Definition of ∞-categories (quasicategories)
   (3) Functors
   (4) Adjunctions
   (5) Mapping spaces
   (6) ∞-category of spaces
   (7) Yoneda
   (8) Homotopy category
   (9) Equivalences in ∞-categories
   (10) Equivalences between ∞-categories
   (11) Maximal subgroupoid
   (12) Geometric realization
   (13) Limit constructions

1.2. **Stable ∞-categories.**
   (1) Definition of stable ∞-categories [Lur09, Lur16a]
   (2) Triangulated structure on the homotopy category
   (3) Stabilization of ∞-categories
   (4) Example: derived category of abelian category
   (5) $K_0$ of a stable ∞-category

1.3. **Spectra.** See also [Gep19], [Lur16a], [YD19]
   (1) Definition of spectra and ring spectra
   (2) Stabilization
   (3) Mapping spectra in stable ∞-categories
   (4) Compact objects
   (5) $\text{Mod}_R, \text{Mod}^{\text{perf}}_R, \text{Mod}^{\text{proj}}_R$ for ring spectra $R$
   (6) Connection to (simplicial) commutative rings

1.4. **$S_\bullet$-construction and $K_0$.**
   (1) Waldhausen $S_\bullet$ construction on pointed ∞-category [Lur14, §14]
   (2) Example: K-theory of ring spectra via projective modules and via perfect modules [Lur14, §19]
   (3) Relation to $K_0$: $K_0$ by hand
   (4) Relation to $BGL(A)^+$
   (5) Example: $K_0$(finite pointed spaces)

1.5. **Reduction to K-theory of spectra.** See also [BGT10]
   (1) Additivity theorem [Lur14, §17]
   (2) Invariance under stabilization
   (3) Ind-objects and idempotent completion [Lur14, §15], [Lur06]
   (4) $K(\text{Ind}(\mathcal{C}))$ for stable ∞-categories $\mathcal{C}$
(5) $K$-theory of pointed $\infty$-categories with finite colimits in terms of $K$-theory of ring spectra $\text{End}(C)$ [Lur14, §19]

1.6. **Additive and Localizing invariants.**

1. Presentable $\infty$-cats
2. Verdier quotients of presentable stable $\infty$-categories
3. (Split) exact sequences of stable $\infty$-categories
4. Additive invariants [BGT10]
5. $K$-theory is the universal additive invariant

1.7. **Non-connective $K$-theory.**

1. Localizing invariants
2. Non-connective $K$-theory
3. Universal property
4. $K$-theory of open immersion of schemes
5. Weibel’s conjecture
6. Cofinality theorem and connection to connective $K$-theory

1.8. **Connections to classical $K$-theory.**

1. Exact 1-categories (Or Waldhausen, depending on what is easier to connect to the previous)
2. $K$-theory of exact 1-categories
3. Theorem of the heart
4. Classical results: Excision, Mayer-Vietoris, ... [Wei13, §5]

2. **DAG**

2.1. **Higher topos theory.**

1. Grothendieck topologies
2. $\infty$-sites
3. Left-exact localization functors
4. Groupoid objects
5. $\infty$-topoi
6. Example of $\infty$-topoi: $\infty$-sheaves on $\infty$-sites
7. $\infty$-stacks on $\text{Sh}(T)$ are equivalently $\infty$-sheaves on $T$, for $\infty$-topoi $T$

2.2. **Derived schemes.** We can follow [Kha16], [Mat12]

1. Derived rings, the associated $E_\infty$-rings, the underlying discrete rings
2. Open immersions of derived affine schemes
3. Zariski topology on $\text{sRing}$
4. Open immersions of Zariski stacks
5. Derived schemes

2.3. **Derived Stacks.** We can follow [Kha16], [Mat12]

1. Cotangent complex
2. Étale topology
3. Derived étale stacks
4. Quasi-coherent modules on stacks: connective and non-connective, the case of derived schemes
5. Smooth morphisms of derived schemes
6. Derived Artin stacks
2.4. Perfect complexes and $K$-theory of stacks.  
(1) Quasi-coherent sheaves on stacks, Artin stacks, schemes  
(2) The adjunction $f^* \dashv f_*$ for a map of stacks $f : X \to Y$  
(3) Perfect complexes on derived stacks [BZFN10], [Lur16b], Khan lecture notes  
(4) Symmetric monoidal $\infty$-categories: dualizable objects  
(5) Compact objects  
(6) The relations between perfect complexes, dualizable object and compact object in $\text{QCoh}(X)$  
(7) $f^*$ preserves perfect complexes  
(8) $\text{Perf}(X)$ is stable $\infty$-cat

2.5. $K$-theory of projective bundles.  
(1) Locally free modules of finite rank  
(2) Projective bundles on stacks and Serre’s twisting sheaf [Kha18b, §6]  
(3) Semi-orthogonal decompositions of stable $\infty$-categories [Kha18b], [YD19]  
(4) Additive invariants ‘split’ semi-orthogonal decompositions [Kha18b, Lem. 2.3.3]  
(5) Semi-orthogonal decomposition on $\text{Perf}(\mathcal{P}(E))$

(1) Quasi-smooth immersions of derived Artin stacks  
(2) Derived blow-ups in quasi-smooth centers  
(3) Semi-orthogonal decomposition on $\text{Perf}(\text{Bl}_Z X)$

2.7. Zariski Descent. See [Kha18b, §§3,4]  
(1) Quasi-compact open immersions of derived schemes  
(2) qcqs derived schemes  
(3) Perfect complexes that vanish on open subsets  
(4) Zariski descent for squares  
(5) Zariski descent for Čech covers

2.8. Grothendieck-Riemann-Roch I. See [Kha18a, §9]  
(1) Derived Sym and $\Lambda$  
(2) Additive $K$-theory (earlier?)  
(3) $\lambda$-rings, $\gamma$-operations, associated grading  
(4) $\lambda$-ring structure on $K_0(R) = K^e_0(\text{Mod}_R^{\text{pro}})$  
(5) Global resolution property  
(6) Globalization of $\lambda$-ring structure to $K_0(X)$ for derived scheme $X$

2.9. Grothendieck-Riemann-Roch II. See [Kha18a, §10]  
(1) Formulation of GRR-theorem for derived schemes

REFERENCES


